AC-2003

Seat No.________

Second Year B. C. A. Examination

April / May – 2003

Mathematical Foundation of Computer Science

Time :   Hours] [Total Marks : **50**

Instructions : (1) **All** questions are **compulsory.**
(2) Figures to the **right** indicate **full** marks.

1 (a) Using the statements : **4**

p : I am in a good mood.
q : I will go to a movie.
r : I will study discrete mathematics.

Write the following statements in symbolic form :

(i) If I am not in a good mood, then I will go to a movie.
(ii) I will not go to movie and I will study discrete mathematics.
(iii) I will go to movie if I will not study discrete mathematics.
(iv) If I neither go to a movie nor study discrete mathematics then I am in a good mood.

(b) Attempt any **two** : **6**

(i) Without using truth tables establish the equivalence of :

$$(p\vee q)\wedge\left(\neg P\wedge\left(\neg P\wedge q\right)\right)\Leftrightarrow\left(\neg P\wedge q\right)$$

(ii) Obtain principal conjunctive normal form and principal disjunctive normal form of

$$(P\vee Q)\wedge\left(\neg P\vee Q\vee R\right)\wedge Q$$

(iii) Draw logic circuit corresponding to $\left(\overline{a+bc}\right)+b$.

2 (a) Define a binary relation on a set. Given $S=\{1,2,3,....10\}$ and a relation R on S where $R=\left\{<x,y>\mid x+y=10\right\}$. Write members of R; Relation matrix of R and draw the graph of the relation R. **4**

(b) Attempt any **two** : **6**

(I) For the lattice $(L,*,\oplus)$ and $a,b,c\in L$. Show that if $a\le b\le c$,

(i) $a\oplus b=b*c$ (ii) $(a*b)\oplus(b*c)=(a\oplus b)*(a\oplus c)$.

(II) Show that in a distributive lattice whenever complement of an element exists then it must be unique.

(III) Draw the Hasse diagram of (S_{16}, D). Is it distributive and complemented ? – Justify.

3 (a) Define atoms, anti-atoms in a Boolean Algebra. In the power-set lattice $\left(P(s), \cap, \cup, \sim, \phi, S\right)$. What are the atoms and anti-atoms if $S = \{a, b, c\}$. **4**

(b) Attempt any **two** : **6**

(I) Define a subboolean algebra. Draw Hasse diagram of $\langle S_{42}, D\rangle$ and find all the subboolean algebras of $\langle S_{42}, D\rangle$.

(II) Prove that for a Boolean Algebra $\left(B, *, \oplus, ', 0, 1\right)$ the following identities hold :

(i) $a * \left(a' \oplus b\right) = a * b$ (ii) $\left(a * b\right) \oplus \left(a * b'\right) = a$

(iii) $(a * b * c) \oplus (a * b) = a * b$.

(III) Define :

(i) Adjacency Matrix (ii) Path Matrix

(iii) Leaf of a tree.

4 (a) For the square matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 1 \end{bmatrix}$ verify that **4**

$A \cdot \left(adj\, A\right) = |A| \cdot I = \left(adj\, A\right) \cdot A$.

(b) Attempt any **two** : **6**

(i) Solve the following equations by Matrix Inversion method :

$x + y = 0$
$y + z = 1$
$z + x = 3$

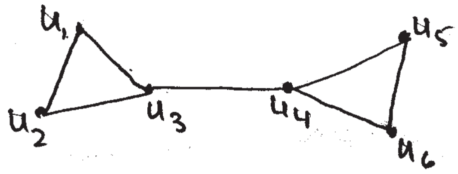
(ii) Define the rank of a matrix. Determine the rank of the following matrix :

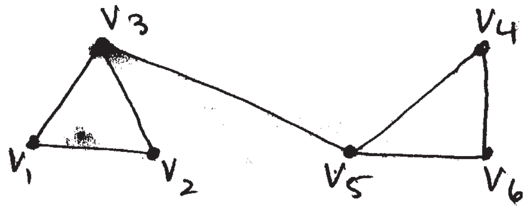
$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

(iii) f $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ then find the matrices $P = \tfrac{1}{2}(A + A')$ and $Q = \tfrac{1}{2}(A - A')$. Now express A as a sum of symmetric and skew-symmetric matrix.

5 (a) When are two simple digraphs said to be isomorphic ? **4**

Find out whether the below two graphs are isomorphic or not.

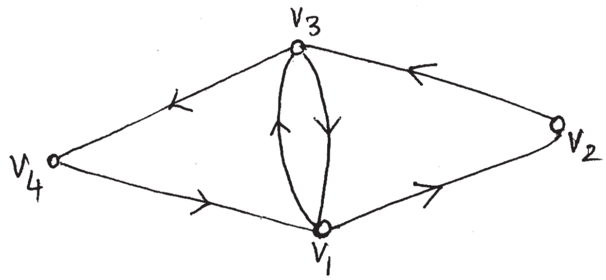




(b) Attempt any **two** : **6**

(i) Define a node base and state the properties of a node base.

(ii) Define strong component of a digraph. Find strong components of the following digraph :



(iii) Represent the following tree as a binary tree :