

**H-56076**

Seat No. \_\_\_\_\_

**M. Sc. (Part - II) Examination**

April / May – 2003

**Statistics : Paper - VII**

**(Industrial Statistics & Operations Research)**

Time : 3 Hours]

[Total Marks : 75

- Instructions :** (1) All questions carry **equal** marks.  
(2) Use of scientific calculator and statistical tables is permitted.

- 1** (a) Define the terms : Reliability, mtbf, expected termination time in connection with life testing theory.  
(b) Explain type-II censoring scheme. For exponential life time model with mean failure time  $\theta, \theta > 0$ . Obtain mle of  $\theta$  and  $R(t)$  under type-II censoring without replacement when  $n$  items are put up to test.  
(c) In connection with part (b) derive UMPU test for testing  $H: \theta = \theta_0$  versus  $K: \theta \neq \theta_0$ . How can you use this test to derive  $100(1-\alpha)\%$  UMAU confidence interval for reliability  $R(t)$  at time  $t$ .

**OR**

- 1** (a) Obtain an expression for probability density function as a function of hazard function. Hence show that if hazard function is constant then the life time model is exponential.  
(b) State the continuous life time model which possesses the memoryless property and show that it is unique.  
(c) Discuss the structure function of a system. Draw the block diagram of a system having the following structure function

$$\phi(\underline{X}) = X_5 [1 - (1 - X_1)(1 - X_2 X_3)(1 - X_4 X_5)] X_6$$

Hence deduce an expression for the system reliability. What will be the expression if all the components are equireliable ?

- 2** (a) What is the difference between Shewhart control chart and Cusum control chart ? Explain the tabular Cusum chart for monitoring process variability.

- (b) Explain Cusum control chart suggested by Johnson. How will you calculate ARL for this chart ?
- (c) Derive *Johnson's* Cusum chart for number of defects per unit. Also obtain ARL of the chart.

**OR**

- 2**
- (a) Define exponential weighted moving average (EWMA) for individual observations. Show that EWMA is a weighted average of all previous sample means. Derive variance of EWMA.
  - (b) An EWMA control chart uses  $\lambda = 0.4$ . How wide will the limits be on the Shewhart control chart expressed as a multiple of the width of the steady state EWMA limits.
  - (c) What is process capability ratio (PCR) ? Discuss about the PCR for an offcenter process. State the third generation version of PCR. If PCR = 1.53 what will be your interpretation about the process ?

- 3**
- (a) Derive economic batch quantity for a deterministic manufacturer's model without shortages. Also obtain an expression for its optimum cycle time.
  - (b) An automobile factory manufactures a particular type of gear within the factory. The gear is used in the final assembly. The particulars of this gear are as follows :  
 Demand rate  $r = 14000$  units per year  
 Production rate  $k = 35000$  units per year  
 Set up cost = Rs. 500 per set-up.  
 Carrying cost = Rs. 15 per unit per year.  
 Find economic batch quantity and cycle time.
  - (c) Define queue and explain briefly the phases of theory of queues.

**OR**

- 3**
- (a) Explain the queueing model  $M/M/k/\infty/FIFO$ . For it derive difference differential equations and obtain their steady state solution.

- (b) At a central warehouse, vehicles arrive at the rate of 18 per year in a Poisson fashion. The unloading time of the vehicles follow exponential distribution and unloading rate is 6 vehicles per hour. There are four unloading crews. For this system find  $P_0, P_3, L_q, L_s, W_q$  and  $W_s$ .
- (c) Write a short note on EOQ models.

- 4 (a) Write a short note on :  
Group replacement policy and individual replacement policy.
- (b) The failure rates of 1000 street bulbs in a colony are summarized as below :

End of Month	1	2	3	4	5	6
Probability of failure to date	0.05	0.20	0.40	0.65	0.85	1

The cost of replacing on individual bulb is Rs. 60. If all the bulbs are replaced simultaneously it would cost Rs. 25 per bulb. Find out whether individual replacement policy or group replacement policy is optimal. Hence find at what equal interval should all bulbs be replaced if group replacement policy is optimal.

**OR**

- 4 (a) Discuss the algorithm for  $n$  jobs and 2 machine problem.
- (b) Consider 2 machines and 6 jobs flow shop scheduling problem as shown in below given problem. Obtain the optimal sequence which will minimize the makespan. Also determine the corresponding makespan :

Job I	1	2	3	4	5	6
Machine I	4	10	14	8	18	16
Machine II	6	12	10	12	6	8

- 5 (a) What is simulation ? Discuss Monte Carlo technique for simulation.

- (b) A certain project is to be accomplished and the various activities for this project work have their durations as shown below :

Activity	Work	Duration times (in days)	
	Optimistic	Normal	Pessimistic
1-2	10	12	14
1-3	8	10	12
2-3	7	9	11
2-4	9	10	17
2-5	8	10	18
3-5	5	7	9
4-6	2	4	6
5-4	3	5	7
5-6	4	6	8

- (i) Draw the network diagram.  
(ii) Determine the maximum duration of completion of the project and locate critical path.  
(iii) What is the chance that the project work can be completed. 2 days before the maximum possible duration.

**OR**

- 5 (a) Define the terms :  
(i) Network (ii) Connected graph (iii) Loop (iv) Tree  
(v) PERT and CPM.  
(b) A project consists of a series of tasks labelled  $A, B, \dots, H, I$  with the following relationships ( $W < X, Y$  means  $X$  and  $Y$  can not start until  $W$  is completed,  $X, Y < W$  means  $W$  can not start until both  $X$  and  $Y$  are completed). With this notation construct the network diagram having the following constraints :

$$A < D, E; B, D < F; C < G; B, G < H; F, G < I$$

Find also the minimum time of completion of the project, when the time (in days) of completion of each task is as follows :

Task	A	B	C	D	E	F	G	H	I
Time	23	8	20	16	24	18	19	4	10

Find the total float and free float for each activity.