KB–56097

Seat No._________

M. Sc. (Part - II) Examination

April / May – 2003

Statistics : Paper - VI

(Design of Experiment & Multivariate Analysis)

Time : **3** Hours] [Total Marks : **75**

Instructions : (1) All questions carry **equal** marks.
(2) Use of scientific calculator and statistical tables is permitted.

1 (a) State and prove Gauss–Markoff theorem.

(b) Define : (1) Connectedness (2) Balancedness (3) Orthogonality. Also state necessary and sufficient condition for a Block design to have each of these properties.

(c) Check whether the design with incidence matrix $N = a\,E_{33}$, where a is some positive integer connected, balanced and orthogonal.

OR

1 (a) Define B.I.B.D. $(v,\ b,\ r,\ k,\ \lambda)$. For a BIBD show that (1) $bk = vr$ (2) $r(k-1) = \lambda(v-1)$ (3) $b \geq v$.

(b) Define SBIBD $(v = b,\ r = k, \lambda)$ and show that there are λ treatments common between any two blocks.

(c) Obtain Intra–block analysis of variance of a BIBD $(v,\ b,\ r,\ k,\ \lambda)$.

2 (a) Define a PBIBD with m–associate classes. Show that in a PBIBD with m associate classes :

(1) $n_i\ p^i_{jk} = n_j\ p^j_{ik} = n_k\ p^k_{ij}$

(2) $\sum_{i=1}^{m} n_i\ \lambda_i = r(k-1)$.

(b) Define a Youden square design and construct it using SBIBD $(v = b = 5,\ r = k = 4,\ \lambda = 3)$.

(c) Explain the terms :

(1) Galois Field

(2) Finite Euclidean Geometry.

OR

2 (a) Define : (1) Main effect and (2) K–factor interaction for 2^m factorial experiment. Discuss the procedure for confounding K independent interactions in 2^m factorial Designs.

(b) Show that contrasts representing two different interactions are orthogonal.

(c) Construct Mutually Orthogonal Latin Squares (MOLS) L_1 of side 8.

3 (a) Describe situations where a Fractional replicate is recommended. Construct a half replicate of 2^6 factorial experiment taking defining contrast as ABCDEF.

(b) Construct a 3^3 confounded factorial design in 9 blocks of 3 plots each by confounding pencils (P(101) and P(112)).

(c) Construct PBIBD using a CUBE.

OR

3 (a) Define canonical correlation coefficients and canonical variates. In usual notation show that the canonical correlations are solution of the determinant equation :

$$\begin{vmatrix} -\lambda\Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & -\lambda\Sigma_{22} \end{vmatrix} = 0$$

Further show that multiple correlation coefficient and simple correlation coefficient are special cases of canonical correlations.

(b) Obtain null distribution of the sample correlation coefficient matrix $R = (r_{ij})$. Show that $E(|R|) = \prod_{i=1}^{n} \left\{1 - \frac{i-1}{p-1}\right\}$.

(c) Show that partial correlation coefficient between x_1 and x_2 given $\underset{\sim}{x}_3$ is same as the simple conditional correlation coefficient between x_1 and x_2.

4 (a) Define singular and non-singular multivariate normal distribution. Obtain characteristic function of singular multivariate normal distribution.

(b) Let $\underset{\sim}{x}_r (r = 1, 2, \ldots\ldots, k)$ be independently distributed as $N_p(\underset{\sim}{\theta}_r, \Sigma_r)$. Show that for fixed matrices $A_r : m \times p$; $\sum_{r=1}^{k} A_r \underset{\sim}{x}_r$ has m–variate normal distribution with mean vector $\sum_{r=1}^{k} A_r \underset{\sim}{\theta}_r$ and covariance matrix $\sum_{r=1}^{k} A_r \Sigma_r A_r'$. Hence, obtain the distribution of sample mean vector $\bar{\underset{\sim}{x}}$.

(c) If $\underset{\sim}{x}_1, \underset{\sim}{x}_2, \underset{\sim}{x}_3$ are iid as $Np(\underset{\sim}{\mu}, \Sigma)$ random variables and f $\underset{\sim}{y}_1 = \underset{\sim}{x}_1 + \underset{\sim}{x}_2$, $\underset{\sim}{y}_2 = \underset{\sim}{x}_2 - \underset{\sim}{x}_3$; obtain the conditional distribution of y_1 given y_2.

OR

4 (a) Define Wishart matrix V. Obtain the distribution of V when $n \geq p$, $\underset{\sim}{\mu} = \underset{\sim}{0}$ and $\Sigma = I$.

(b) State and prove additivity property of Wishart distribution.

(c) Obtain (i) ED and (ii) $E(ADA')$ where $A: p \times p$ is any nonsingular matrix and $D \sim W_p(D|n|\Sigma)$.

5 (a) Let $\underset{\sim}{x} \sim N_p(\underset{\sim}{\mu}, \Sigma)$. Define James Stien's estimator of mean vector $\underset{\sim}{\mu}$. Show that $d_\alpha(\underset{\sim}{x}) = \left(1 - \frac{\alpha}{\underset{\sim}{x}' \underset{\sim}{x}}\right) \underset{\sim}{x}$ is an admissible estimator for $p \geq 3$ with respect to squared error loss function $l(\underset{\sim}{\mu}, \underset{\sim}{d}) = (\underset{\sim}{d} - \underset{\sim}{\mu})' (\underset{\sim}{d} - \underset{\sim}{\mu})$.

(b) If $\sum = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ where $\rho > 0$, find the first principal component associated with Σ and find the percentage of total variance explained by it.

OR

5 (a) Write a short note on Fisher's linear discriminant.

(b) Discuss the problem of classifying into two unknown multivariate normal population with known common covariance matrix Σ. Obtain explicit expression for error of misclassifications.

(b) Define Dirichlet distribution and Dirichlet integral. Write its applications. If $\underset{\sim}{x}_1, \underset{\sim}{x}_2, \ldots, \underset{\sim}{x}_n$ be an r.s. of size n from the population with pdf $g(\underset{\sim}{x}' A \underset{\sim}{x})$, $|A| > 0$; obtain the pdf of $y = \underset{\sim}{x}' A \underset{\sim}{x}$. If $\underset{\sim}{x} \sim N_p A(\underset{\sim}{0}, \Sigma)$, what is the distribution of $y = \underset{\sim}{x}' \Sigma^{-1} \underset{\sim}{x}$.