56059 Seat No.__________

M. Sc. (Part - II) Examination

April/May – 2003

Statistics : Paper – V

(Statistical Inference)

Time : **3** Hours] [Total Marks : **75**

Instructions : (1) All the questions carry **equal** marks.
(2) Use of scientific calculator and statistical tables is permitted.

1 (a) Describe the main components of a statistical decision problem. Show that the problem of estimation and testing of hypotheses are statistical decision problems.

(b) When do you say a decision rule d_1 is better than d_2 ?
Define :
(1) As good as
(2) Admissible decision rule
(3) Complete class of decision rules
(4) Minimal complete class of decision rules.

(c) Show that if the class of admissible rules is complete, then it is minimal complete.

OR

1 (a) Define :
(1) Loss function and risk function
(2) Non randomised and rundomised decision rules
(3) Prior and posterior distributions
(4) Bayes risk.

(b) Describe Bayes principle and minimax principle for ordering the available decision rules.

(c) Let $\Omega = \{\theta_1, \theta_2, \theta_3\}$; $A = \{a_1, a_2\}$ and the loss function defined on $\Omega \times A$ is

$\theta \backslash a$	a_1	a_2
θ_1	-10	-5
θ_2	-5	-5
θ_3	1	0

and probability distribution is

$\theta \backslash x$	x_1	x_2
θ_1	$1/2$	$1/2$
θ_2	$1/3$	$2/3$
θ_3	$3/4$	$1/4$

Obtain Bayes rule with respect to prior $\pi(\theta)$

$\pi(\theta_1) = 0.5,\ \pi(\theta_2) = 0.3, \pi(\theta_3) = 0.2.$

2 (a) Explain the difference between Games problem and Decision problem, it any.

(b) Define Bayes rule, limit of Bayes rules and generalized Bayes rule.

(c) Using quadratic loss function, obtain Bayes estimator

for θ when : $f(x, \theta) = \dfrac{e^{-\theta}\theta^{x}}{x!}, x = 0, 1, 2.....,\infty,\quad \theta > 0$

and the prior distribution is given by

$g(\theta) = \dfrac{1}{\Gamma\alpha\, \beta^{\alpha}}\, e^{-\theta/\beta}\, \theta^{\alpha-1},\ \theta > 0,\ \alpha, \beta > 0.$

OR

2 (a) Define the risk set of a statistical decision problem. If the parameter space is discrete and finite, prove that risk set S is a convex subset of E_k, where k is the number of elements in the parametric space.

(b) If S is bounded and closed from below and $\pi = (\pi_1, \pi_2, ...\pi_k)$ is a prior distribution over Ω with $\pi_j > 0$ for all j, then prove that Bayes rule with respect to prior π exists.

(c) If the risk set of a decision rule δ_0 is an element of lower boundary set $\lambda(s)$ then show that δ_0 is admissible.

3 (a) State and prove the necessary part of the Neyman Pearson fundamental lemma for randomized test. Why we have to generalize the NP lemma ? State the properties of GNP lemma.

(b) "If both H_0 and H_1 specify the distributions from the same family $\{F_\theta, \theta \in \Omega\}$ and if a sufficient statistic t for θ exists then the BCR is a function of the sufficient statistic. However, the BCR will not be always of the form $t \geq c_\alpha$ or $t \leq c_\alpha$" Examplify the statement.

OR

3 (a) Define the terms : UMPU test, α–similar test, and test with Neyman structure.

(b) Testing the hypothesis $H: \theta = \theta_0$ versus $K: \theta \neq \theta_0$ for exponential family of distribution with pdf (or pmf) $f(x, \theta) = c(\theta) \exp\{\varphi(\theta)\, T(x)\}\, h(x)$, show that for UMPU test $\phi(x)$

$$E_{\theta_0}\{\phi(x)\, T(x)\} = \alpha E_{\theta_0}\{T(x)\}.$$

(c) Let $X \sim f(x, \theta) = \frac{1}{\theta}\, e^{-x/\theta},\ x \geq 0, \theta > 0$. Obtain UMPU test of size α for testing $H := \theta_0$ versus $K: \theta \neq \theta_0$ based on a single observation on X. Also give explicit expression for power function of the test.

4 (a) Prove that for testing the hypothesis $H: \theta \leq \theta_1$ or $\theta \geq \theta_2$, $\theta_1 < \theta_2$ against the alternative $K: \theta_1 < \theta < \theta_2$ in a single parameter exponential family there exists a UMP test given by :

$$\phi(x) = \begin{cases} 1 \ \textbf{if} \ \ c_1 < T(x) < c_2 \\ r_i \ \textbf{if} \ \ T(x) = c_i,\ i = 1, 2 \\ 0 \ \textbf{if} \ \ T(x) < c_1 \ \textbf{or} \ T(x) > c_2 \end{cases}$$

Where the c's and r's are determined by

$$E_{\theta_1}\{\phi(x) = \alpha = E_{\theta_2}\{\phi(x)\}.$$

(b) Let $X \sim P_0(\lambda)$ and $Y \sim P_0(\mu)$, $\lambda > 0$, $\mu > 0$, X and Y are independent. To test $H: \lambda \geq \mu$ versus $K: \lambda < \mu$ derive UMP test of size α. Also obtain the power function of the test.

OR

4 (a) Define uniformly most accurate and uniformly most accurate unbiased confidence intervals.

(b) Let $X_1, X_2, ..., X_n$ be n independent observations from $N(0, \sigma^2)$ distribution to test the hypothesis $H: \sigma = 1$ against $K: \sigma \neq 1$. Obtain UMPU ten of level α. Hence deduce UMAU confidence interval for σ.

(c) Write note on likelihood ratio test and its properties.

5 (a) Let N denote the number of observations required by SPRT with bounds $B < 1 < A$. The show that there exists constants δ and C with $c > 0$ and $0 < \delta < 1$ such that $P(N \geq n) \leq c\delta^n$ Hence show that SPRT terminates eventually with probability one.

(b) Suppose that X be a Bernoulli random variable corresponding to the event E such that p be the probability of success and 1–p be the probability of failure of an event E. Construct an SPRT with strength (α, β) and obtain an expression for the ASN for testing $H: p = p_0$ against $K: p = 1 - p_0$.

(c) What is Mann Whitney U–test ? In what respect does it differ from Wald–wolfowitz runs test ? Explain how one finds the null probability distribution of the Mann–Whitney U–test statistic.

OR

5 (a) Describe Wald–Wolfowitz runs test in case of small and large samples.

(b) Explain chi–square test of goodness of fit. Derive its asymptotic distribution.

(c) State the fundamental identity of sequential probability ratio test. Obtain approximate expressions for (i) O.C. function and (ii) ASN function using the identity.