N–55039 Seat No. ______

M. Sc. (Part - I) Examination

April / May – 2003

Statistics : Paper - III

(Theory of Estimation & Theory of Sampling)

Time : **3** Hours] [Total Marks : **75**

Instructions : (1) All questions carry **equal** marks.

(2) Use of scientific calculator and statistical tables is permitted.

1 (a) State and prove Fisher–Neyman criterion for sufficient statistic. Also show that if the statistics T is sufficient for θ, any monotonic function $\zeta(T)$ will also be sufficient for θ.

(b) Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from $U(0, \theta)$ distribution. Show that $X_{(n)} = \max i \ \{x_1, \ x_2, \ldots\ldots, x_n\}$ is (i) a complete sufficient statistic (ii) minimal sufficient statistic and consistent for θ.

(c) Define minimal sufficient statistic, complete sufficient statistic, ancillary statistic.

Let X be a single observation from the probability distribution

$$f(x/\theta) = \begin{cases} \dfrac{\theta}{2} & \text{if} \quad x = -3 \\ \dfrac{\theta}{3} & \text{if} \quad x = 0 \\ \dfrac{1-2\theta}{3} & \text{if} \quad x = 6, \ 13, \ 52 \\ \theta^2 + \dfrac{\theta}{6} & \text{if} \quad x = 60 \\ \theta - \theta^2 & \text{if} \quad x = 68 \end{cases}$$

where $0 < \theta < \frac{1}{2}$. Find a minimal sufficient statistic for θ and derive the probability distribution of the minimal sufficient statistic.

OR

1 (a) State and prove Lehman–Scheffe theorem and Rao–Blackwell theorem.

(b) Let $x_1, x_2, \ldots\ldots, x_n$ is a random sample from the Poisson distribution with mean $\lambda\,(\lambda > 0)$. Obtain MVUE for

(1) $e^{-2\lambda}$

(2) $e^{-\lambda}(1+\lambda)$

Comment on the estimators.

(c) State Bhattacharya lower bound. One the basis of a random sample of size n obtain Bhattacharya lower pounds L_1 and L_2 for σ in the density

$$f(x,\ \sigma) = \frac{1}{\sigma} e^{-x/\sigma}, \quad x > 0, \quad \sigma > 0.$$

2 (a) Define maximum likelihood estimator. Does mle always exist ? If not, give a counter example. Prove that under regularity conditions, any consistent solution of the likelihood equation provides a maximum of the likelihoods with probability tending to unity as the sample size tends to infinity.

(b) Obtain mle of θ for the distribution having pdf

$f(x;\ \theta) = \frac{1}{2} e^{-|x-\theta|}; \quad -\infty < x, \quad \theta < \infty$. Hence obtain large sample confidence interval for θ based on mle of θ.

(c) Let $x_1, x_2, \ldots\ldots, x_n$ is a random sample taken from $U(\theta,\ \theta+1)$ distribution. Show that $2\,\bar{x}$ is unbiased for θ. Give an expression for estimator that has minimum variance among all unbiased estimators of θ. Also obtain mle of θ.

OR

2 (a) Write note on "The method of maximum likelihood estimation".

(b) Explain the method of minimum chi–square and modified minimum χ^2 estimation.

(c) Consider a population made up of three different types of individuals occurring in the proportions θ^2, $2\theta(1-\theta)$ and $(1-\theta)^2$ respectively, where $0 < \theta < 1$. Let n_1, n_2 and n_3 denote the frequencies of the above three types of individuals. Determine the maximum likelihood estimator of θ and show that it is asymptotically most efficient for θ.

3 (a) For the linear model $\underline{Y} = X\underline{\beta} + \underline{\in}$, $\underline{\in} \sim N(\underline{0},\ \sigma^2 I)$ obtain OLSE of $\underline{\beta}$ and σ^2. Assuming normal distribution for a random variable Y, derive mle of σ^2. Hence show that mle of $\sigma^2 = \left(1 - \frac{k}{n}\right)\left(OLSE \ \ of \ \sigma^2\right)$.

(b) For the model

$$Y_i = \beta_0 + B_1 X_i + \in_i, \quad i = 1,\ 2,\ 3$$

when $X_1 = -1$, $X_2 = 0$, $X_3 = 1$, find that BLUE's of β_0 and β_1.

OR

3 (a) Write notes on any **three** of the following :

(1) Determination of sample size.'

(2) Two stage sampling.

(3) Linear and circular systematic sampling.

(4) Product estimator.

(5) Cluster sampling.

(6) Non–response errors.

4 (a) Give the rationale behind the use of varying probability of selection in surveys. Discuss (i) Cumulative total method (ii) Lahiri's method of selection to obtain PPSWR sample.

(b) Suggest an unbised estimator of population total. Obtain its variance and unbiased estimator of this variance.

OR

4 (a) Discuss Horvitz and Thompson (H.T.) Estimator of population mean. Obtain its variance and estimators of variance suggested by H.T. and Yates and Grandy (Y.G.). Also discuss the practical difficulties in adopting these estimators.

(b) Discuss Des Raj's ordered estimator. Obtain its variance and estimator of variance.

5 (a) Explain the rationale behind the use of ratio method of estimation. When the ratio estimator is expected to be best ?

(b) Discuss Hartly–Ross technique of building unbiased estimator. Use this technique to construct ratio type unbiased estimator of population mean. Obtain its large sample variance and discuss the situations under which it can be better than conventional ratio estimator.

OR

5 (a) Explain the difference estimator in SRSWOR which make use of auxiliary variable. When this estimator reduces to linear regression estimator ? Obtain the variance of the estimator and unbised estimator of this variance.

(b) Discuss double sampling, its need and utility. Discuss double sampling ratio estimator for population mean. Obtain the variance of the estimator and unbiased estimator of this variance.