AA - 3358

Seat No._____

M. Phil. Examination

April / May – 2003

Physics : Paper - I

(Research Methodology)

Time : **3** Hours] [Total Marks :

Instructions : (1) Attempt any **five** questions.
(2) All questions carry **equal** marks.
(3) The symbols have their usual meanings.
(4) Scientific calculator can be allowed.

1 Solve the initial value problem $u' = -2tu^2, u(0) = 1$ with $h = 0.2$ on the interval [0, 1]. Use the second order implicit Runge Kutta method. Obtain $u(0.2)$, $u(0.4)$, $u(0.6)$, $u(0.8)$.

2 Discuss *Gauss-Chebyshev* method for $w(x) = \frac{1}{\sqrt{1-x^2}}$ and use it to evaluate the integration $I = \int_{-1}^{1} (1-x^2)^{3/2} \cos x \, dx$.

3 (a) Discuss the limitation on the χ^2 test.

(b) What is χ^2 test ?

4 Find all the eigen value and eigen vectors of a matrix by *Jacobi* method :

$$\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

5 (a) Find A^{10} when $A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$.

(b) Obtain all the eigenvalues of matrix $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ using the *Rutishauser* method.

6 (a) Use the *Gauss* elimination method to solve following equations :

$10x_1 - x_2 + 2x_3 = 4$

$x_1 + 10x_2 - x_3 = 3$

$2x_1 + 3x_2 + 20x_3 = 7.$

(b) Use the decomposition method to solve the following equation :

$x_1 + x_2 + x_3 = 1$

$4x_1 + 3x_2 - x_3 = 6$

$3x_1 + 5x_2 + 3x_3 = 4$

7 Prove that :

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{r} \frac{\left(n_{ij} - \frac{n_i \, n_j}{n} \right)^2}{\frac{n_i \, n_j}{n}}$$

8 Discuss the methods based on finite difference operators for numerical differentiation.

9 (a) Evaluate the integral $I = \int_1^2 \int_1^2 \frac{dxdy}{x+y}$ using the trapezoidal rule with $h = k = 0.5$ and $h = k = 0.25$.

(b) Find the Jacobian matrix for the system of equations

$f_1(x, y) = x^2 + y^2 - x = 0$

$f_2(x, y) = x^2 - y^2 - y = 0$

with $h = k = 1$

10 (a) Find the condition number (k) of the system

$$\begin{bmatrix} 2.1 & 1.8 \\ 6.2 & 5.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 6.2 \end{bmatrix}$$

(b) Find the inverse of the coefficient matrix of the system

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

by the *Gauss-Jordan* method with partial pivoting and hence solve the system.