53823 Seat No._____

First Year B. C. A. Examination

April / May – 2003

Advanced Mathematics

Time : Hours] [Total Marks : **50**

Instructions : (i) Figures to the **right** indicate **full** marks.

(ii) Each question carries **10** marks.

(iii) Use of calculator is allowed provided they are silent and battery operated.

1 (a) Let U be a universal set and A and B be two non-empty sets of U. Prove the DeMorgan's law **4**

$$(A \cap B)' = A' \cup B'$$

(b) Attempt any **two** parts :

(i) If $A = \{0,\ 1,\ 2,\ 3,\ 4\}$, $B = \left\{x \middle| x^2 + x - 6 = 0\right.$ and $x \in A\}$ and $C = \left\{x \middle| x^2 + x - 12 = 0 \text{ and } x \in A\right\}$ **6**

then verify $A \cup (B \cup C) = (A \cap B) \cup (A \cap C)$.

(ii) The population of a town is 6000. Out of these 3400 persons read Hindustan Times and 2700 persons read The Times of India. There are 700 persons who read both the papers. Find the number of persons who donot read any of these two papers. Illustrate through Venn-diagram.

(iii) Given the function $f(x) = 2x + 3$, show that

$$f(2x) - 2f(x) + 3 = 0.$$

2 (a) Find maximum and minimum value of the function : **4**

$$f(x) = 2x^3 + 9x^2 - 60x + 25$$

(b) Attempt any **two** parts : **6**

(i) Evaluate the following :

(1) $\lim_{x \to 5} \frac{x^3 - 125}{x^2 - 25}$

(2) $\lim_{n \to \infty} \left(1 + \frac{5}{n}\right)^n$

(ii) When is the function $f(x)$ said to be continuous at $x = a$? Check the continuity of the function defined as

$$f(x) = \frac{3}{2}ax - (a+1) \qquad x \neq 2$$
$$= 1 \qquad x = 2$$

If $f(x)$ is continuous at $x = 2$ what should be the value of a ?

(iii) If $e^x + e^y = x + y$ then prove that

$$\frac{dy}{dx} = \frac{1 - e^x}{e^y - 1}.$$

3 (a) Find the area of the region bounded by $y = x^2$ and the line $y = x + 2$. **4**

(b) Evaluate the following integrals : (any **three**) **6**

(i) $\int_0^{\pi/2} \sin^5 x \, dx$

(ii) $\int_3^5 (x+3)^3 \, dx$

(iii) $\int e^x \sin x \, dx$

(iv) $\int_2^4 \frac{1}{25 - x^2} \, dx$

(v) $\int \frac{3x + 2}{(x-2)(x-3)} \, dx$

4 (a) Find the equation of a straight line which makes intercepts of a and b on x-axis and y-axis respectively. **4**

(b) Attempt any **two** parts : **6**

(i) Find the area of the quadrilateral whose vertices are $A(1, 1)$, $B(3, 4)$, $C(5, -2)$ and $d(4, -7)$.

(ii) Find the equation of lines passing through the intersection of $4x - 3y - 1 = 0$ and $2x - 5y + 3 = 0$ and parallel to $4x + 5y = 6$.

(iii) In what ratio is the line joining $(3, 7)$ and $(6, 3)$ divided by the line joining $(9, 0)$ and $(17, -10)$?

5 (a) Obtain the order and degree of the following differential equations : (any **two**) **4**

(i) $(4x+3)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

(ii) $\sqrt{\frac{d^2y}{dx^2}}=2\frac{dy}{dx}$

(iii) $\left(\frac{d^3y}{dx^3}\right)^3+\left(\frac{d^2y}{dx^2}\right)^4+3y=0$

(b) Solve the following differential equations : (any **two**) **6**

(i) $xy^2\frac{dy}{dx}=x^3+y^3$

(ii) $\frac{dy}{dx}+2y=e^{-x}$

(iii) $(2x+3y+5)dx+(3x+5y+7)\,dy=0.$