56857 Seat No.__________

B. Sc. Fire (Technology) Examination

April / May – 2003

Applied Mathematics

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All questions are **compulsory**.
(2) Figures to the **right** indicate **full** marks of the question.
(3) Non–programmable scientific calculators are permitted.
(4) Assume suitable additional data, that may be necessary.

1 (a) A line makes angles α, β, γ and with δ the diagonals of a cube. Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$. **5**

OR

(a) If $\left|\vec{A} + \vec{B}\right| = \left|\vec{A} - \vec{B}\right|$ find the angle between $\vec{A}$ and $\vec{B}$. **5**

(b) Attempt any **three** from the following : **9**

(1) Define cross product of two vectors. Explain properties of it.

(2) Constant forces $\vec{P} = 2\hat{i} - 5\hat{j} + 6\hat{k}$ and $\vec{Q} = -\hat{i} + 2\hat{j} - \hat{k}$, acting on a particle. Determine the work done when the particle is displaced from A to B. The position vectors of A and B being $4\hat{i} - 3\hat{j} - 2\hat{k}$ and $6\hat{i} + \hat{j} - 3\hat{k}$.

(3) Find a unit vector, parallel to the sum of vectors $\vec{R_1} = 2\hat{i} + 4\hat{j} - 5\hat{k}$, $\vec{R_2} = \hat{i} + 2\hat{j} + 3\hat{k}$.

(4) If $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{B} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{C} = 3\hat{i} + \hat{j}$, find t such that $\left(\vec{A} + t\vec{B}\right)$ is perpendicular to $\vec{C}$.

(5) Explain : Direction cosines.

(6) Using vector analysis, prove that $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$.

2 (a) (1) Give definition of convergence of a series and state the necessary condition for convergence of positive terms series. **2**

(2) Examine the convergence of the series $1+2+3+........+n+........+\infty$ **2**

OR

(a) Give statement and proof for the convergence of "Geometric series." **5**

(b) Test for convergence any **three** series from the following : **9**

(1) $\sum_{n=0}^{\infty} \frac{3^{2n}}{2^{3n}}$

(2) $\sum_{n=1}^{\infty} \frac{1}{1+2^{-n}}$

(3) $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{n}} + ...\infty$

(4) $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + ...\infty$

3 (a) State and prove : Leibnitz's theorem, for the n^{th} derivative of the product of two functions. **5**

OR

(a) State and prove : "Cauchy's Mean–Value Theorem". **5**

(b) Attempt any **three** : **9**

(1) If $x = a\cos^3\theta$, $y = b\sin^3\theta$. find y_2.

(2) If $y = e^{ax}\sin bx$, prove that $y_2 - 2ay_1 + \left(a^2 + b^2\right)y = 0$

(3) If $y = \left(\sin^{-1}x\right)^2$, show that

$$\left(1 - x^2\right)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$$

(4) If $ax^2 + 2hxy + by^2 = 1$, prove that

$$y_2 = \frac{h^2 - ab}{(hx + by)^3}.$$

(5) Using Maclaurin's series, expand $\tan x$, upto term containing x^5.

(6) Explain e^x in powers of $(x-1)$ upto four terms.

4 (a) Explain Homogeneous functions. State and prove Euler's theorem on homogeneous functions. **6**

OR

(a) Define : Increasing and decreasing functions. Give one example of each function. **6**

(b) Attempt any **two** : **8**

(1) If $\sin u = \frac{x^2 y^2}{x + y}$, then using Euler's theorem prove that, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$.

(2) If $f(x, y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2 + y^2}$, then by using Euler's theorem prove that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + 2f(x, y) = 0$.

(3) For that values of x, the function $x^3 - 3x^2 - 9x + 22$ is :

(1) Increasing

(2) Decreasing

(3) Stationary

Also find stationary values of the function.

(4) Find the maximum and minimum values of $x^5 - 5x^4 + 5x^3 - 1$.

(5) Find the centre of gravity of a uniform hollow cone of height h.

5 (a) Evaluate the following integers : (any **three**) **12**

(1) $\int \left(\frac{\cos x}{1+\cos x}\right) dx$

(2) $\int \left(\frac{x^5 - 1}{x - 1}\right) dx$

(3) $\int \left(\frac{x^3}{x^2 - a^2}\right) dx, \left(x^2 > a^2\right)$

(4) $\int \left(x^2 e^{3x}\right) dx$

(5) $\int \left(\frac{\sin 5x}{\sin x}\right) dx$

(b) Find the area of the region bounded by the circle $x^2 + y^2 = r^2$ **2**

OR

(b) Prove that the area of the region enclosed by **2**

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; (a > b)$ is πab.